Possibility of large lifetime differences in neutral B meson systems

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Abstract

We investigate new physics models that can increase the lifetime differences in the B_q–$\overline{B}_q$ systems $(q = d, s)$ above their standard model values. If both B_q as well as $\overline{B}_q$ can decay to a final state through flavour dependent new physics interactions, the so-called Grossman bound may be evaded. As examples, we consider the scalar leptoquark model and λ''-type R-parity violating supersymmetry. We find that models with a scalar leptoquark can enhance $\Delta\Gamma_s/\Gamma_s$ all the way up to its experimental upper bound and $\Delta\Gamma_d/\Gamma_d$ to as much as $\sim 2.5\%$, at the same time allowing the CP violating phase β_s to vary between $-45°$ and $20°$. R-parity violating supersymmetry models cannot enhance the lifetime differences significantly, but can enhance the value of β_s up to $\sim \pm 20°$. This may bring the values of $\Delta\Gamma_q/\Gamma_q$ as well as β_s within the measurement capabilities of B factories and LHCb. We also obtain bounds on combinations of these new physics couplings, and predict enhanced branching ratios of $B_{s/d} \to \tau^+\tau^-$.

I. INTRODUCTION

The standard model (SM) has been successful in explaining almost all the observations at the accelerator experiments so far, which makes it a challenge to look for signals of new physics. Apart from the direct searches for new particles at colliders, tests of the low energy predictions of the SM can also provide indirect signatures of the physics beyond the standard model (BSM). In the domain of flavour physics in particular, such low energy observables include the branching ratios of various B decay modes, the extent of CP violation in these decays, as well as the oscillation parameters of neutral B meson systems [1]. The data from the B factories and the Tevatron have already played a crucial role in constraining the nature and extent of BSM physics.

In this paper we shall concentrate on the oscillation parameters in the B_d–$\overline{B}_d$ as well as B_s–$\overline{B}_s$ systems. For convenience, we shall refer to the labels d and s collectively as q. The average lifetimes $\overline{\Gamma}_q \equiv (\Gamma_{qH} + \Gamma_{qL})/2$, mass differences $\Delta M_q \equiv M_{qH} - M_{qL}$, lifetime differences $\Delta\Gamma_q \equiv \Gamma_{qL} - \Gamma_{qH}$, as well as CP asymmetries $\sin 2\beta_q$ with $\beta_q^{\rm SM} \equiv {\rm Arg}[-(V_{cb}^* V_{cq})/(V_{tb}^* V_{tq})]$ offer incisive probes of new physics. Here the labels L and H stand respectively for the light and heavy mass eigenstates in the neutral B_q system. The values of $\Gamma_q, \Delta m_q$ and $\sin 2\beta_d$ have already been measured to an accuracy of better than $\sim 5\%$ [2, 3, 4] and play an important role in constraining any new physics. The remaining quantities, on the other hand, currently have large errors and their accurate measurements act as tests of the SM. The value of $\sin 2\beta_s$, for example, which is predicted to be ≈ -0.03 in the SM, can be enhanced significantly with many BSM physics models [5] and a measurement in excess of the SM prediction would vouch for the presence of new physics.

The SM predicts the lifetime differences in the B_d and B_s system to be [6] $\Delta\Gamma_d/\Gamma_d = (0.41^{+0.09}_{-0.10})\%$ and $\Delta\Gamma_s/\Gamma_s = (14.7 \pm 6.0)\%$ respectively, $\Delta\Gamma_d$ being suppressed with respect to $\Delta\Gamma_s$ by a factor of $\sim |V_{td}/V_{ts}|^2 \approx 0.05$. The measurement of the latter is within the capability of LHCb, whereas that of the former is very difficult even at the super-B factories due to its extremely small value. The large theoretical uncertainties in the SM predictions for these two quantities make them rather unsuitable for the detection of BSM physics, unless such physics changes the value of $\Delta\Gamma_q/\Gamma_q$ beyond the SM uncertainties. The so-called "Grossman theorem" states that new physics can only decrease the value of $\Delta\Gamma_s$ [7], and the result extended to the B_d system [8] implies that $\Delta\Gamma_d$ can increase at the most by

20% with BSM contributions. This would seem to make the measurements of $\Delta\Gamma_q$ rather unappealing from the point of view of detecting new physics.

However, the Grossman theorem, and its extension mentioned above, are applicable only when the BSM physics contributes to the dispersive part M_{12q} of the B_q–$\overline{B}_q$ mixing amplitude, and not to its absorptive part Γ_{12q}. This is true for most of the BSM models, in particular the minimal flavour violation (MFV) models [9] where the CP violation emerges only from the CKM matrix. Even in non-MFV models, if the mixing box diagram contains only heavy degrees of freedom, BSM physics cannot contribute to Γ_{12q}. Such models include R-parity conserving supersymmetry, models with universal extra dimensions, little Higgs models, two-Higgs doublet models, etc. On the other hand, there are well-motivated models where the B_q–$\overline{B}_q$ mixing box diagram contains two light degrees of freedom, resulting in an absorptive amplitude. We will discuss two such examples in this paper: (i) models with a scalar leptoquark, and (ii) R-parity violating (RPV) supersymmetry. These models can have *flavour dependent* couplings of light known particles with one heavy new particle (squark or leptoquark), and hence can contribute to Γ_{12q}. This paves the way for an evasion of the Grossman bound, and potentially high lifetime differences in both the B_s and B_d systems. We emphasize that these models are chosen just as examples and by no means exhaust the list of all such possible models.

The new physics couplings also contribute to the mixing amplitudes (hence the mass splittings ΔM_q between the stationary states and the CP asymmetries), and to decay rates. As a consequence, the BSM parameter space is severely constrained by these data. In spite of these constraints, we show that these BSM models can indeed enhance the lifetime difference in the B_s up to its current experimental limit, obtained from the angular analysis of $B_s \to J/\psi\phi$ decays [10, 11]. In the B_d system, the value of $\Delta\Gamma_d/\Gamma_d$ can become as much as 2.5% and hence come within the capabilities of the B factories [12]. As a bonus, the mixing-induced CP asymmetry $\sin 2\beta_s$ can be enhanced by an order of magnitude from its small SM value. As an important byproduct, we also obtain limits on the couplings of the BSM models considered above, and predict enhanced branching ratios for decay channels correlated with the enhanced lifetime differences.

The rest of the paper is organised as follows. In Sec. II, we clarify the definitions, methodology and approximations used to calculate the SM as well as BSM contributions to M_{12q} and Γ_{12q}. In Sec. III, we present our numerical results, which give us limits on

the new physics couplings, as well as the enhancements of lifetime differences, β_s, and rates of correlated decay modes of B_q. Sec. IV summarises our findings and discusses their implications for B physics experiments.

II. THE FORMALISM

A. The Standard Model

The effective Hamiltonian for the B_q–$\overline{B}_q$ system in the flavour basis, with CPT conservation, is given by

$$H_{eff} = \begin{pmatrix} M_{11q} - \frac{i}{2}\Gamma_{11q} & M_{12q} - \frac{i}{2}\Gamma_{12q} \\ M_{12q}^* - \frac{i}{2}\Gamma_{12q}^* & M_{11q} - \frac{i}{2}\Gamma_{11q} \end{pmatrix} . \tag{1}$$

With the approximation $|\Gamma_{12q}| \ll |M_{12q}|$, which is valid for the B_d as well as the B_s system, we have [8]

$$\Delta M_q \approx 2|M_{12q}| , \qquad \Delta\Gamma_q \approx \frac{2\mathrm{Re}(M_{12q}\Gamma_{12q}^*)}{|M_{12q}|} = 2|\Gamma_{12q}|\cos\Phi_q , \tag{2}$$

where Φ_q is the phase difference between M_{12q} and Γ_{12q}. This in fact demonstrates the Grossman theorem for B_s: since $\Phi_s \approx 0$ in the SM, as long as there is no BSM contribution to Γ_{12s} itself, the value of $\Delta\Gamma_s$ will always decrease due to the cosine factor in (2).

We now indicate the methodology of our calculations. In the SM, the dispersive part of the B_q–$\overline{B}_q$ mixing amplitude is given by [13]

$$M_{12q}^{\mathrm{SM}} = \frac{G_F^2}{12\pi^2}\hat{\eta}_{B_q} M_{B_q} B_{B_q} f_{B_q}^2 M_W^2 (V_{tb}^* V_{tq})^2 S_0(x_t) , \tag{3}$$

where G_F is the Fermi constant, M_X is the mass of particle X, and V_{ij}s are the CKM matrix elements. The short distance behaviour is contained in $\hat{\eta}_{B_q}$, which incorporates the QCD corrections, and in the Inami-Lim function

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3} , \tag{4}$$

with x_f for a fermion f defined by

$$x_f \equiv m_f^2/M_W^2 . \tag{5}$$

The decay constant f_{B_q} and the bag factor B_{B_q} take care of the hadronic matrix element

$$\langle \overline{B}_q|(\overline{b}q)_{V-A}(\overline{b}q)_{V-A}|B_q\rangle = (8/3)M_{B_q}^2 f_{B_q}^2 B_{B_q} , \tag{6}$$

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with the wavefunction for the B_q meson normalised as

$$2M_{B_q}|M_{12q}| = \langle \overline{B_q}|H_{eff}|B_q \rangle \ . \tag{7}$$

The absorptive part of the B_q–$\overline{B}_q$ mixing amplitude in the SM, to leading order in QCD, is given by [13, 14]

$$\Gamma_{12q}^{\text{SM}(0)} = -\frac{G_F^2 f_{B_q}^2 B_{B_q} M_{B_q}}{8\pi}(V_{cb}^* V_{cq})^2 m_b^2 F(c) \ , \tag{8}$$

where

$$F(f) = \sqrt{1 - 4\frac{m_f^2}{m_b^2}} \left(1 - \frac{2}{3}\frac{m_f^2}{m_b^2}\right) \tag{9}$$

for a fermion f. The next to leading order QCD corrections and the $1/m_b$ corrections modify the value of Γ_{12s} [15] as well as Γ_{12d} [8] in the SM by $\sim 30\%$. The current theoretical status of $\Delta\Gamma_q/\Gamma_q$ has been summarised in [6].

In the presence of any BSM contribution, we have

$$M_{12q} = M_{12q}^{\text{SM}} + M_{12q}^{\text{BSM}} \ , \qquad \Gamma_{12q} = \Gamma_{12q}^{\text{SM}} + \Gamma_{12q}^{\text{BSM}} \ . \tag{10}$$

In our numerical analysis in Sec. III, we use the SM predictions [6] that include the NLO QCD and $1/m_b$ corrections, however for $\Gamma_{12q}^{\text{BSM}}$ we only use the leading order contributions $\Gamma_{12q}^{\text{BSM}(0)}$. Note that the QCD corrections are expected to be different for SM and BSM operators since the mediating heavy particle for the latter case is a colour triplet. The $1/m_b$ corrections are also expected to differ since the light degrees of freedom that flow inside the mixing box are different too. While it is desirable to have an idea of these corrections, in this work we will just assume that these corrections are likely to introduce an error of $\sim 30\%$ in our calculations. Since our final results claim enhancements of up to 5 times over the lifetime differences in the SM, and the BSM model calculations themselves depend on the unknown masses of the new particles, the higher order corrections would not change the conclusions qualitatively.

B. Leptoquark Models

Leptoquarks are colour-triplet objects that couple to quarks and leptons. They occur generically in GUTs [16], composite models [17], and superstring-inspired E_6 models [18]. Model-independent constraints on their properties are available [19], and the prospects of

their discovery at the LHC have also been studied [20]. We shall restrict ourselves to scalar leptoquarks that are singlets under the $SU(2)_L$ gauge group of the SM. This is because vector or $SU(2)_L$ nonsinglet leptoquarks tend to couple directly to neutrinos, hence we expect that their couplings are tightly constrained from the neutrino mass and mixing data. This makes any significant effect on the B_q–$\overline{B}_q$ system unlikely.

The relevant interaction term for a scalar leptoquark S_0 is of the form

$$\mathcal{L}_{\mathrm{LQ}} = \lambda_{ij}\overline{d^c}_{jR}e_{iR}S_0 + \mathrm{h.c.} \;, \tag{11}$$

where d_R and e_R stand for the right-handed down-type quarks and right-handed charged leptons respectively, and i, j are generation indices that run from 1 to 3. The couplings λ_{ij} can in general be complex, and some of them may vanish depending on any flavour symmetries involved.

When λ_{32} and λ_{33} are nonzero, the interaction (11) generates an effective four-fermion $(S+P) \otimes (S+P)$ interaction leading to $b \to s\tau^+\tau^-$. This will contribute to B_s–$\overline{B}_s$ mixing (with τ and S^0 flowing inside the box), to the leptonic decay $B_s \to \tau^+\tau^-$, and to the semileptonic decays $B_q \to X_s\tau^+\tau^-$. The relevant quantity here is the coupling product

$$h_{LQ}(b \to s\tau^+\tau^-) \equiv \lambda_{32}^*\lambda_{33} \;, \tag{12}$$

such that $h_{LQ}(b \to s\tau^+\tau^-)/(8M_{S_0}^2)$ is the effective leptoquark coupling equivalent to $(G_F/\sqrt{2})V_{tq}^*V_{tb}$ or $(G_F/\sqrt{2})V_{cq}^*V_{cb}$ in the SM.

By changing the leptonic index from 3 to 2, one gets the second generation leptoquark that can lead to $b \to s\mu^+\mu^-$ with the relevant coupling product

$$h_{LQ}(b \to s\mu^+\mu^-) \equiv \lambda_{22}^*\lambda_{23} \;. \tag{13}$$

In addition to B_s–$\overline{B}_s$ mixing, the coupling $h_{LQ}(b \to s\mu^+\mu^-)$ contributes also to $B_s \to \mu^+\mu^-$ and $B_q \to X_s\mu^+\mu^-$. The upper bound on the branching ratio of $B_s \to \mu^+\mu^-$ constrains this coupling product severely, as will be seen in Sec. III. One can also have mixed leptonic indices, giving rise to the channel $b \to s\tau^+\mu^-$ for example, which we will not discuss here. One expects the first generation leptoquarks to be heavier, from the Tevatron and HERA data [21], so the couplings of the first generation leptoquarks are highly constrained, and we do not consider them in this paper. However, note that the bounds are, in general, dependent on the quantum numbers of the leptoquarks.

The B_d–$\overline{B}_d$ mixing is modified by the coupling product

$$h_{LQ}(b \to d\tau^+\tau^-) = \lambda_{31}^*\lambda_{33} , \qquad h_{LQ}(b \to d\mu^+\mu^-) = \lambda_{21}^*\lambda_{23} . \tag{14}$$

The former combination $h_{LQ}(b \to d\tau^+\tau^-)$ affects $B_d \to \tau^+\tau^-$ and $B_d \to X_d\tau^+\tau^-$, whereas the latter affects $B_d \to \mu^+\mu^-$ and $B_d \to X_d\mu^+\mu^-$.

In terms of the coupling products h_{LQ}, the contributions of the leptoquarks to the B_q–$\overline{B}_q$ mixing is given by

$$M_{12q}^{\mathrm{LQ}} = \sum_{\ell=\mu,\tau} \frac{h_{LQ}^2(b \to q\ell^+\ell^-)}{384\pi^2 M_{S_0}^2} M_{B_q}\hat{\eta}_{B_q}f_{B_q}^2 B_{B_q}\tilde{S}_0(x_\ell) ,$$

$$\Gamma_{12q}^{\mathrm{LQ}(0)} = -\sum_{\ell=\mu,\tau} \frac{h_{LQ}^2(b \to q\ell^+\ell^-)}{256\pi M_{S_0}^4} M_{B_q}f_{B_q}^2 B_{B_q}m_b^2 F(\ell) . \tag{15}$$

where the function $\tilde{S}_0(x)$ is

$$\tilde{S}_0(x) = \frac{1+x}{(1-x)^2} + \frac{2x\log x}{(1-x)^3} , \tag{16}$$

and $F(f)$ is as given in (9). While calculating the limits on the new physics parameter space, we assume that at a time, only one of $h_{LQ}(b \to q\tau^+\tau^-)$ and $h_{LQ}(b \to q\mu^+\mu^-)$ is nonvanishing, so that the right hand side of each of the equation in (15) has only one term.

As noted above, leptoquarks contribute to the leptonic decay $B_q \to \ell^+\ell^-$. In the SM, this decay rate is extremely small [22]: $BR(B_d \to \mu^+\mu^-) \approx 1.1 \times 10^{-10}$, $BR(B_d \to \tau^+\tau^-) \approx 3.1 \times 10^{-8}$. We therefore approximate the branching fraction of $B_q \to \ell^+\ell^-$ by only the leptoquark contribution [23]:

$$\mathrm{BR}(B_q \to \ell^+\ell^-) \approx \frac{|h_{LQ}(b \to q\ell^+\ell^-)|^2}{128\pi M_{S_0}^4}\frac{f_{B_q}^2 M_{B_q}^3}{\Gamma_{B_q}}\frac{m_\ell^2}{M_{B_q}^2}\sqrt{1 - 4\frac{m_\ell^2}{M_{B_q}^2}} . \tag{17}$$

The presence of leptoquarks may be detected through the measurement of such an enhanced branching fraction. On the other hand, upper bounds on these branching fractions constrain the leptoquark coupling products discussed above.

C.   R-parity violating supersymmetry (RPV SUSY)

R-parity is the discrete symmetry defined as $R = (-1)^{3B+L+2S}$, where B, L, and S are respectively baryon number, lepton number and spin of a particle. R equals 1 for all SM

particles and -1 for all superparticles. Though R-parity is conserved *ad hoc* in a large class of supersymmetric models, one can write R-parity violating terms respecting Lorentz and gauge invariance. These terms can violate either B or L, but not both, since that will lead to uncontrollably large proton decay rate. In this work, we consider only the B-violating terms. Most of the L-violating λ'-type couplings are highly constrained from neutrino mass [24] and ΔM_s measurement [25], so their contribution to the neutral meson mixing is expected to be small.

Consider the B violating term in the superpotential

$$\mathcal{W} = \epsilon_{\alpha\beta\gamma} \lambda''_{ijk} U^c_{i\alpha} D^c_{j\beta} D^c_{k\gamma} \,, \tag{18}$$

where α, β, γ are colour indices. The couplings λ''_{ijk} are in general complex, and are antisymmetric in the last two generation indices j and k, *i.e.*, $\lambda''_{ijk} = -\lambda''_{ikj}$. In terms of the component fields, this can give rise to terms in the Lagrangian that are of the form

$$\epsilon_{\alpha\beta\gamma} \lambda''_{ijk} \tilde{u}^c_{i\alpha} \overline{d}_{j\beta} P_R d^c_{k\gamma} \,, \quad \text{or} \quad \epsilon_{\alpha\beta\gamma} \lambda''_{ijk} \overline{u}_{i\alpha} P_R d^c_{k\gamma} \tilde{d}^c_{j\beta} \,, \tag{19}$$

where $P_R = (1 + \gamma_5)/2$. As $j \neq k$, the only possible combinations responsible for B_s–$\overline{B}_s$ mixing are $\lambda''_{i13} \lambda''_{i12}{}^*$. For B_d–$\overline{B}_d$ mixing, the relevant combination is $\lambda''_{i23} \lambda''_{i21}{}^*$. As usual, we consider only one product to be nonzero at a time. This also helps us avoid the tighter constraints coming from, say, $K^0 - \overline{K}^0$ mixing.

Let us first consider B_s–$\overline{B}_s$ mixing. The λ'' couplings with $i = 1$ are highly constrained to be at most $\sim \mathcal{O}(10^{-4} - 10^{-5})$ from n-$\overline{n}$ oscillation and double nucleon decay [26]. Though there is no significant bound on the $i = 2$ couplings, the corresponding coupling products, if comparable with the SM, would contribute and modify $b \to c\overline{c}s$ processes. Just to avoid this consequence, we do not consider the $i = 2$ couplings any further and move to nonzero values of $i = 3$ couplings. This leads to two new mixing diagrams as shown in Fig. 1: one with internal d quarks and $\tilde{t}$ squarks, and the other with internal t quarks and $\tilde{d}$ squarks. While both these diagrams contribute to M_{12s}, only the former has an absorptive contribution that goes towards Γ_{12s}. This leads to

$$
\begin{aligned}
M_{12s}^{\mathrm{RPV}} &= \frac{h_{\mathrm{RPV}}^2(b \to s)}{192\pi^2 M_{\tilde{q}_R}^2} M_{B_s} \hat{\eta}_{B_s} f_{B_s}^2 B_{B_s} \left(\tilde{S}_0(x_t) + \tilde{S}_0(x_d) \right) \,, \\
\Gamma_{12s}^{\mathrm{RPV}(0)} &= -\frac{h_{\mathrm{RPV}}^2(b \to s)}{128\pi M_{\tilde{q}_R}^4} M_{B_s} f_{B_s}^2 B_{B_s} m_b^2 F(d) \,,
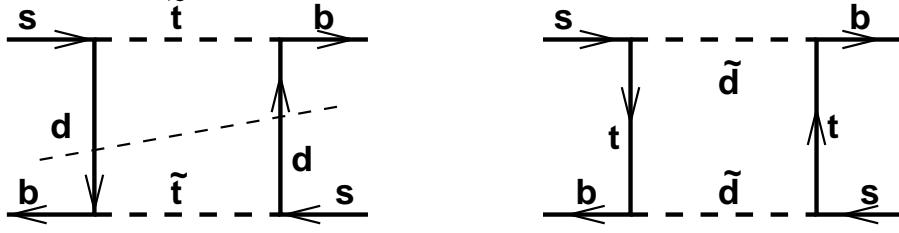\end{aligned}
\tag{20}
$$

FIG. 1: B_s–$\overline{B}_s$ mixing diagrams with RPV SUSY. Note that only the first diagram gives an absorptive part. To get the total amplitude, one must also include the crossed boxes.

where

$$h_{\mathrm{RPV}}(b \to s) \equiv \lambda''^{*}_{313}\lambda''_{312} \tag{21}$$

is the relevant coupling product in RPV SUSY, while the factors $\tilde{S}_0(x)$ and $F(d/s)$ are as given in (16) and (9) respectively. We have assumed the relevant squarks, $\tilde{t}_R$ and $\tilde{d}_R$, to be degenerate in mass.

The coupling product $h_{\mathrm{RPV}}(b \to s)$ also contributes to nonleptonic B decays taking place via $b \to d\bar{d}s$, like $B^+ \to K^0\pi^+, B^+ \to K^+\pi^0, B_d \to K^0\pi^0, B_s \to \phi\pi^0, B_s \to \pi^+\pi^-$ and their CP conjugate decays. The decay rates depend on the coherent sum of the SM and the BSM amplitudes. The former is calculated in the naive factorisation model [27]. However, considering the uncertainties in any such calculation, we have been slightly conservative: the form factors are not directly taken from [27] but fitted so that even without the BSM part, the pure SM expectation of the branching ratio is consistent with the data. This means that the branching ratio data may not be used for claiming the presence of BSM physics; it can, at best, constrain the parameter space from above. The BSM part is computed with some further simplifying assumptions. We assume the strong phase difference to be zero between the SM and the BSM amplitudes. However, since the weak phase of the BSM coupling is varied over its full range, this effect is offset for B^+ decays. We have also not taken into account the mixing between the RPV operators and the SM operators between the scales M_W and m_b. The dominant effect, which is just a multiplicative renormalisation of the RPV operator, can be taken into account by redefining the couplings so that their values are calculated at the scale m_b and not at the high scale (thus, one should be careful in using the constraints on the couplings; to get the constraints at a higher scale, one must run them upwards).

The B_d–$\overline{B}_d$ system may be analyzed in an analogous manner, simply with the interchange

$d \leftrightarrow s$. With the naive factorization approximation, The mixing amplitude has the dispersive and the absorptive parts

$$M_{12d}^{\mathrm{RPV}} = \frac{h_{\mathrm{RPV}}^2(b \to d)}{192\pi^2 M_{\tilde{q}_R}^2} M_{B_d} \hat{\eta}_{B_d} f_{B_d}^2 B_{B_d} \left(\tilde{S}_0(x_t) + \tilde{S}_0(x_s) \right),$$

$$\Gamma_{12d}^{\mathrm{RPV}(0)} = -\frac{h_{\mathrm{RPV}}^2(b \to d)}{128\pi M_{\tilde{q}_R}^4} M_{B_d} f_{B_d}^2 B_{B_d} m_b^2 F(s) , \qquad (22)$$

where the coupling product involved is

$$h_{\mathrm{RPV}}(b \to d) \equiv \lambda_{323}''^* \lambda_{321}'' , \qquad (23)$$

and the notation used is the same as in the B_s–$\overline{B}_s$ case. One may note that in a number of well-motivated models, the lighter stop may be significantly lighter than the other squarks. In this case, the value of $\Delta\Gamma_s$ ($\Delta\Gamma_d$) would be higher than the one computed here.

The coupling product $h_{\mathrm{RPV}}(b \to d)$ also contributes to the nonleptonic $b \to s\overline{s}d$ decay channels like $B^{+,0} \to \pi^{+,0}\phi$. Indeed, the measurement of $\mathrm{BR}(B^+ \to \phi\pi^+)$ provides the strongest constraint on $|h_{\mathrm{RPV}}(b \to d)|$.

Note that the relations for RPV SUSY contain a relative factor of two compared to the leptoquark case; this is due to the fact that in RPV, the effective $|\Delta B| = 1$ Hamiltonian contribution to $b \to d\overline{d}s$ ($b \to s\overline{s}d$) in the B_s (B_d) system contains two terms, which come from the contraction of two colour ϵ-factors in (19): $\epsilon_{abc}\epsilon_{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$ where δ is the Kronecker delta function.

III. NUMERICAL RESULTS

In this section we shall numerically evaluate the allowed values of $\Delta\Gamma_q/\Gamma_q$ and $\sin 2\beta_s$ in the framework of the BSM models considered in the previous section, applying the constraints from current measurements. Unless otherwise mentioned, all numbers are taken from [28]. The average lifetimes of B_q mesons,

$$\Gamma_d = (0.653 \pm 0.004) \; \mathrm{ps}^{-1} , \qquad \Gamma_s = (0.682 \pm 0.027) \; \mathrm{ps}^{-1} , \qquad (24)$$

are not affected much by the BSM physics. The measured values of the mass differences are

$$\Delta M_d = (0.507 \pm 0.004) \; \mathrm{ps}^{-1} , \qquad \Delta M_s = (17.33_{-0.21}^{+0.42} \pm 0.07) \; \mathrm{ps}^{-1} . \qquad (25)$$

The value of $\sin 2\beta_d$ in the SM, obtained from a fit that does not involve B_d–$\overline{B}_d$ mixing, is [4]

$$\sin 2\beta_d^{\text{SM}} = 0.752 \pm 0.040 \tag{26}$$

whereas the CP asymmetry in the B_d–$\overline{B}_d$ system is measured, from charmonium modes, to be [4]

$$\sin 2\beta_d = 0.674 \pm 0.026 \ . \tag{27}$$

We constrain the BSM physics by requiring $\sin 2\beta_d$ to lie in this interval. The current limits on the values of $\Delta\Gamma_q$ and β_s [4, 28, 29],

$$\frac{\Delta\Gamma_s}{\Gamma_s} = 0.206^{+0.106}_{-0.111} \ , \quad \frac{\Delta\Gamma_d}{\Gamma_d} = 0.009 \pm 0.037 \ , \quad \beta_s = -0.79 \pm 0.56 \ , \tag{28}$$

though very weak, are also included for consistency. In particular, it will be seen that in a model with third generation leptoquark, the possible value of $\Delta\Gamma_s/\Gamma_s$, which should lie in $(-0.01, 0.51)$ at 95% C.L. [1] [28], is bounded only by the experimental limit.

We also use the values of the bag factors [4, 30]

$$f_{B_d}\sqrt{B_{B_d}} = 223 \pm 35 \text{ MeV} \ , \qquad f_{B_s}\sqrt{B_{B_s}} = 262 \pm 35 \text{ MeV} \ , \tag{29}$$

and the short distance factors

$$\hat{\eta}_{B_d} = \eta_{B_s} = 0.55 \ , \qquad S_0(x_t) = 2.327 \pm 0.044 \ . \tag{30}$$

The relevant CKM elements are

$$|V_{td}| = 8.54(28) \times 10^{-3} \ , \qquad |V_{ts}| = 40.96(61) \times 10^{-3} \ , \tag{31}$$

while the other elements are taken to be fixed at their central values. For leptonic decays, we use $f_{B_s} = f_{B_d} = 200$ MeV.

To constrain the BSM effects, we present our results taking the SM contribution to $\Delta\Gamma_q$ to be $\Delta\Gamma_s = (0.096 \pm 0.039)$ ps^{-1} and $\Delta\Gamma_d = (26.7 \pm 6.1) \times 10^{-4}$ ps^{-1} [6]. The SM theoretical uncertainty is about 30% for $\Delta\Gamma_s$ and about 25% for $\Delta\Gamma_d$, so one must be careful

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[1] The likelihood distribution of $\Delta\Gamma_s/\Gamma_s$ is extremely skewed [28], so that the upper limit of the confidence interval is rather high compared to the naive 2σ estimate. Note that we have used the Heavy Flavor Averaging Group (HFAG) limits obtained from a fit to only the direct measurements of the lifetime difference. If combined with the lifetimes obtained from flavour specific modes and $B_s \to K^+ K^-$, the 95% C.L. upper bound on $\Delta\Gamma_s/\Gamma_s$ will decrease to 0.25 [28].

in interpreting the results. Only when the values of $\Delta\Gamma_q$ in the presence of BSM physics exceed the SM calculations beyond their upper limit including the large error bars can the presence of new physics be claimed. As will be seen in the following, it is indeed possible in the scalar leptoquark model with a third generation leptoquark.

The new physics models are parametrised by the magnitude of the relevant coupling product h_{BSM} and its weak phase. We vary these two as free parameters and scan the parameter space, taking the SM parameters to have Gaussian distributions with the means and standard deviations as given above. We require the calculated quantities to be within the 95% C.L. (2σ) intervals of the experimental measurements, where such a measurement is available (e.g. ΔM_q, $\Delta\Gamma_q$). Whenever only an upper bound is available (e.g. some branching ratios), we require the calculated quantities to be within the 90% C.L. interval, since these are the quoted limits [28].

A. Leptoquark

As pointed out in Sec. II B, the couplings that may enhance $\Delta\Gamma_q/\Gamma_q$ also tend to enhance the branching ratio $B_q \to \ell^+\ell^-$, where ℓ is the charged lepton of the corresponding leptoquark generation. The limits coming from the measurements of these decay modes therefore constrain the allowed ranges of BSM parameters. The relevant branching ratios are [28]

$$\text{BR}(B_d \to \mu^+\mu^-) < 2.3 \times 10^{-8} \, , \qquad \text{BR}(B_d \to \tau^+\tau^-) < 4.1 \times 10^{-3} \, ,$$
$$\text{BR}(B_s \to \mu^+\mu^-) < 7.5 \times 10^{-8} \, . \tag{32}$$

The leptoquark mass is taken to be 100 GeV.

The severe constraints on $B_q \to \mu^+\mu^-$ restricts the coupling products of the second generation leptoquark to $|h_{LQ}(b \to d\mu^+\mu^-)| < 4 \times 10^{-4}$ and $|h_{LQ}(b \to s\mu^+\mu^-)| < 6 \times 10^{-4}$, as a consequence there is no significant enhancement in either $\Delta\Gamma_q/\Gamma_q$ or β_s. Since the couplings of the third generation leptoquark are not so severely restricted, it is possible to enhance the values of these quantities.

We show in Fig. 2 our results in the B_s system with the third generation leptoquark. We display the allowed values of the phase of the coupling product, $\text{Arg}[h_{LQ}(b \to s\tau^+\tau^-)]$, as well as the allowed values of $\Delta\Gamma_s/\Gamma_s$, β_s and $\text{BR}(B_s \to \tau^+\tau^-)$, as functions of the magnitude of the coupling product, $|h_{LQ}(b \to s\tau^+\tau^-)|$. We observe the following:
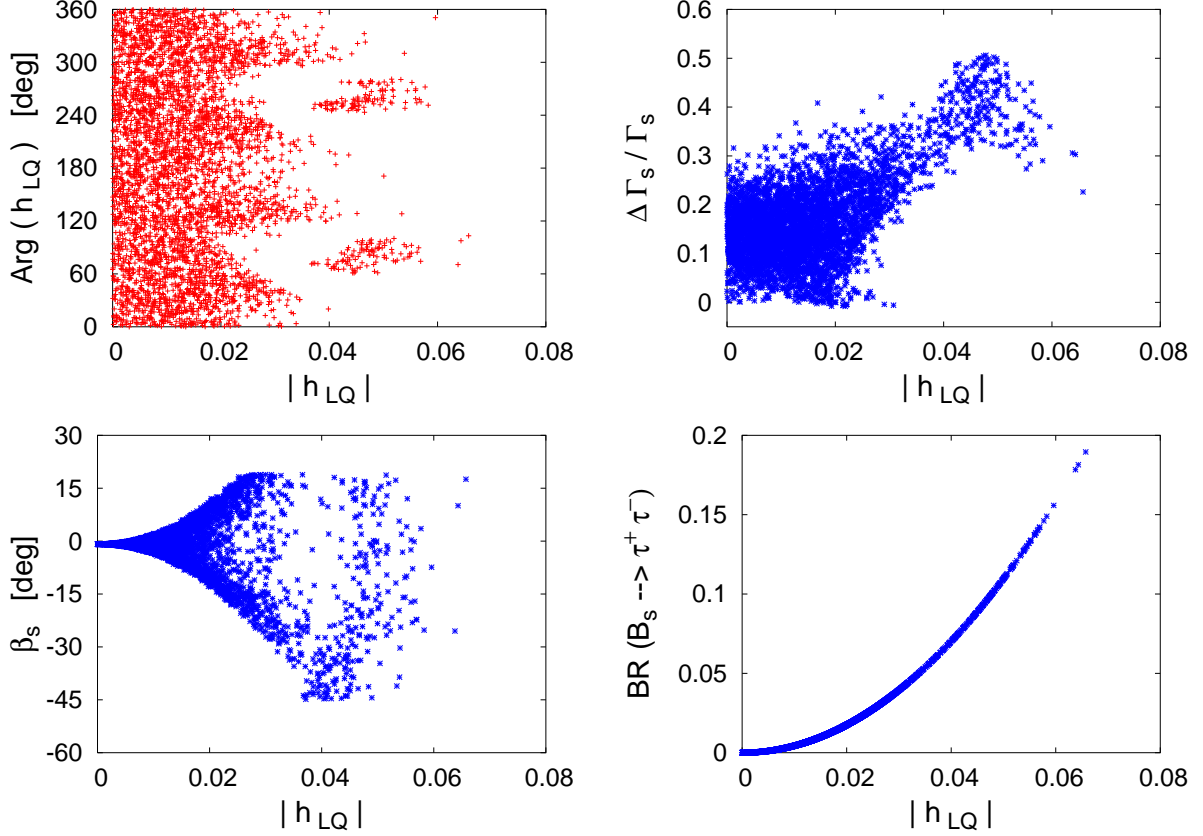
FIG. 2: The allowed values of the phase of the coupling product, $\text{Arg}[h_{LQ}(b \to s\tau^+\tau^-)]$, as well as the corresponding allowed values of $\Delta\Gamma_s/\Gamma_s$, β_s and $\text{BR}(B_s \to \tau^+\tau^-)$ as functions of the magnitude of the coupling product, $|h_{LQ}(b \to s\tau^+\tau^-)|$. The mass of the leptoquark is taken to be 100 GeV. Note that we have used the 95% C.L. bounds on $\Delta\Gamma_s/\Gamma_s$ from only its direct measurements.

- The major constraints on the parameter space of h_{LQ} arise from the ΔM_s measurement (25). The allowed parameter space is not continuous, but has small islands at higher values of $|h_{LQ}|$. This is due to the constructive (destructive) interference between the SM and the BSM amplitudes for ΔM_s, which forbids (allows) some values of $\text{Arg}(h_{LQ})$ with the ΔM_s measurements.

- The value of $\Delta\Gamma_s/\Gamma_s$ is bounded from above only by the current experimental limit (28), which is more than three times the central value predicted by the SM, and more than 5σ away from it even when the theoretical uncertainties are included. If indeed a third generation leptoquark of mass ~ 100 GeV is present, the measurement of $\Delta\Gamma_s/\Gamma_s$ is literally round the corner, perhaps even possible at the Tevatron [32].

- The value of β_s can become as high as $20°$ or as low as $-45°$, which makes its measure-

ment with the decay modes like $B_s \to J/\psi\eta^{(')}$, or through the angular distributions of $B_s \to J/\psi\phi$ [10] or $B_s \to D_s^{+(*)}D_s^{-(*)}$ [11], easily feasible at the B factories or at the LHC experiments [31] .

- A large value of $\Delta\Gamma_s/\Gamma_s$ is also accompanied by a large branching ratio $B_s \to \tau^+\tau^-$, which may be enhanced to as much as 18%. Currently no measurement of this decay channel is available, however if it indeed has such a large decay rate, its measurement would also indicate a large value of $\Delta\Gamma_s$.

Analogous results with the B_d–$\overline{B}_d$ system are displayed in Fig. 3, where we show the allowed parameter space of the magnitude and phase of $h_{LQ}(b \to d\ell^+\ell^-)$, as well as the corresponding allowed $\Delta\Gamma_d/\Gamma_d$ values and their correlations with $\mathrm{BR}(B_d \to \tau^+\tau^-)$. Here, the mass difference ΔM_d as well as the well measured value of $\sin 2\beta_d$ (27) restrict the leptoquark coupling. It may be seen from the figure that:

- A rather specific value of the relative phase between the SM and the BSM amplitudes allows one to obey the experimental constraints while at the same time allowing for the value of $\Delta\Gamma_d/\Gamma_d$ as high as 2.5%.

- Such a high lifetime difference also comes in conjunction with a $B_d \to \tau^+\tau^-$ branching ratio as high as 0.4%, which is just below the current experimental upper bound.
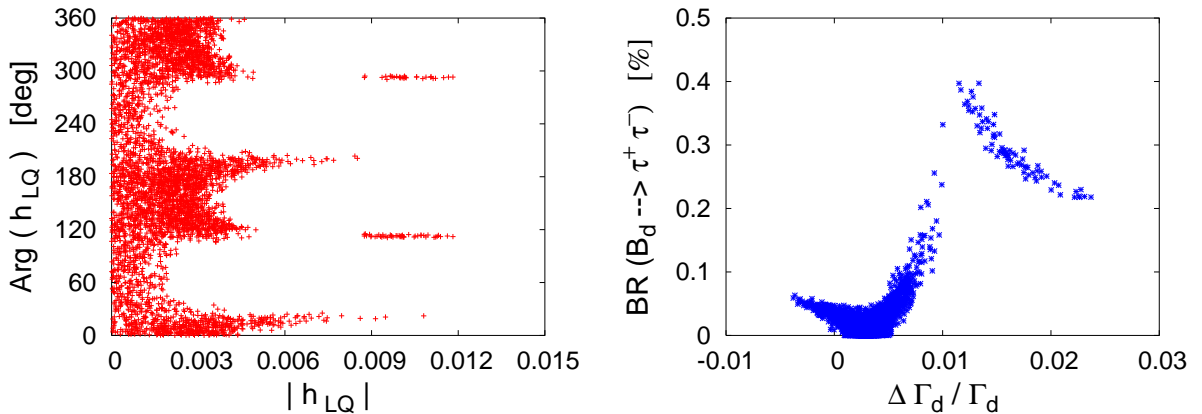

FIG. 3:   The left panel shows the allowed parameter space of the phase and magnitude of the coupling product $h_{LQ}(b \to d\tau^+\tau^-)$. The right panel shows the allowed values of $\Delta\Gamma_d/\Gamma_d$ along with their correlation with $\mathrm{BR}(B_d \to \tau^+\tau^-)$. The mass of the leptoquark has been taken to be 100 GeV.

Thus, the mediation due to a third generation scalar leptoquark is able to increase $\Delta\Gamma_d/\Gamma_d$ to a value large enough to be measurable at the B factories [12]. This should be a strong motivation for trying to measure this quantity in the B_d system. At the same time, the measurement of $\mathrm{BR}(B_d \to \tau^+\tau^-)$ should also be able to indicate whether $\Delta\Gamma_d/\Gamma_d$ is significant or not.

B. R-parity violating supersymmetry

The R-parity violating couplings that may enhance $\Delta\Gamma_q/\Gamma_q$ also tend to enhance the branching ratios $B_d \to \phi\pi^0$ and $B^+ \to K^0\pi^+, \phi\pi^+$. The relevant measurements are [28]

$$\mathrm{BR}(B^+ \to \phi\pi^+) < 0.24 \times 10^{-6} , \qquad \mathrm{BR}(B^+ \to K^0\pi^+) = (23.1 \pm 1.0) \times 10^{-6} ,$$

$$\mathrm{BR}(B^0 \to \phi\pi^0) < 0.28 \times 10^{-6} . \tag{33}$$

Taking all squarks to be degenerate at 300 GeV, the values of $\Delta\Gamma_s/\Gamma_s$ and β_s consistent with the above branching ratios and the ΔM_s measurement are shown in Fig. 4. Note that for the form factor in the $B^+ \to K^0\pi^+$ mode, we use the value of the form factor that reproduces the central value of the $\mathrm{BR}(B^+ \to K^0\pi^+)$ measurement in (33) with naive factorization, i.e. we assume that the current measurement shows no new physics effects. This makes our new physics estimates rather conservative.
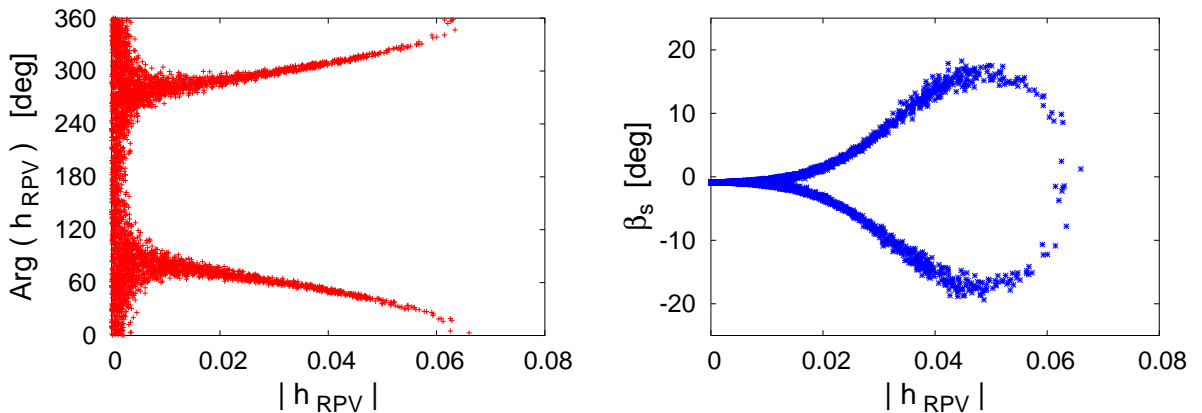

FIG. 4: The left panel shows the allowed parameter space of the phase and the magnitude of the coupling product $h_{\mathrm{RPV}}(b \to s)$. The right panel shows the allowed values of β_s as a function of $|h_{\mathrm{RPV}}(b \to s)|$. The masses of the right handed squarks $\tilde{t}_R$ and $\tilde{d}_R$ have been taken to be 300 GeV.

Clearly, the value of $\Delta\Gamma_s/\Gamma_s$ in this scenario does not increase much beyond its SM value. However, the CP violating phase β_s may be enhanced to as much as $\pm 20°$, making the CP violation observable through the decay modes $B_s \to J/\psi\phi$, $B_s \to D_s^{+(*)}D_s^{-(*)}$, or $B_s \to J/\psi\eta^{(')}$ [32].

Note that at large $|h_{\mathrm{RPV}}|$, values of $\mathrm{Arg}(h_{\mathrm{RPV}}) \approx 0$ are preferred, whereas $\mathrm{Arg}(h_{\mathrm{RPV}}) \approx \pi$ is strongly disfavoured. This indicates that for $\mathrm{Arg}(h_{\mathrm{RPV}}) \approx 0$ (π), the SM and RPV amplitudes to ΔM_s interfere destructively (constructively). This is because the effective four-Fermi Hamiltonian for the R-parity violation model comes with an opposite sign to that of the SM Hamiltonian. This may be explained as follows. The $(S - P) \times (S + P)$ interaction in the RPV SUSY gives $-(1/2)(V + A) \times (V - A)$ under Fierz reordering, and when the charge-conjugated spinors are replaced by ordinary spinors, a flip of their positions, which also changes the $V - A$ Lorentz structure to $V + A$, gives another negative sign. On top of that, the internal propagator is scalar and not a vector gauge particle, which introduces another negative sign in the amplitude. The BSM contribution thus has a net negative sign compared to the SM value, when the relative weak phase between the two amplitudes is zero.

The RPV SUSY scenario does not give rise to any significant new physics effects in the B_d–$\overline{B}_d$ system, since the CP violating phase β_d is also well measured, and in combination with the limit on $\mathrm{BR}(B \to \phi\pi)$, constrains the new physics parameter space severely.

IV. SUMMARY AND CONCLUSIONS

We have studied models with (i) scalar leptoquark and (ii) R-parity violating supersymmetry as examples of BSM physics that may enhance the values of the lifetime differences $\Delta\Gamma_s/\Gamma_s$ as well as $\Delta\Gamma_d/\Gamma_d$. Such an enhancement is possible since the new couplings in these models are flavour dependent, and there are additional light degrees of freedom in the B_q–$\overline{B}_q$ mixing box diagram. The latter contribute to Γ_{12q}, thus enabling the evasion of the so-called Grossman bound, so that the lifetime differences can be higher. Since the values of $\Delta\Gamma_q/\Gamma_q$ in the SM have uncertainties of $\sim 30\%$, we look for enhancements of $\mathcal{O}(1)$, so that such a large $\Delta\Gamma_q/\Gamma_q$ would be a clean signal of new physics.

We summarise our results in Table IV. Let us point out the salient features that emerge from our analysis.

Model	$|h_{\mathrm{BSM}}|$ bound	$\Delta\Gamma_q/\Gamma_q$ range	β_q range	Related branching ratios
B_s–$\overline{B}_s$				
LQ, 2nd gen.	6×10^{-4}	$(-0.01, 0.35)$	$\approx -1°$	$\mathrm{BR}(B_s \to \mu^+\mu^-) < 7.5 \times 10^{-8}$
LQ, 3rd gen.	0.07	$(-0.01, 0.51)$	$(-45°, 20°)$	$\mathrm{BR}(B_s \to \tau^+\tau^-) \to 18\%$
RPV SUSY	0.065	$(-0.01, 0.35)$	$(-20°, 20°)$	$\mathrm{BR}(B \to K\pi) \approx 2.3 \times 10^{-5}$
B_d–$\overline{B}_d$		[in %]		
LQ, 2nd gen.	4×10^{-4}	$(0.1, 0.5)$	$\approx \beta_d$ (exp)	$\mathrm{BR}(B_d \to \mu^+\mu^-) < 2.3 \times 10^{-8}$
LQ, 3rd gen.	0.012	$(-0.5, 2.5)$	$\approx \beta_d$ (exp)	$\mathrm{BR}(B_d \to \tau^+\tau^-) \to 0.4\%$
RPV SUSY	3.2×10^{-3}	$(0.1, 0.5)$	$\approx \beta_d$ (exp)	$\mathrm{BR}(B_d \to \phi\pi) < 0.25 \times 10^{-6}$

TABLE I: Allowed values of the magnitude of the coupling product, $|h_{\mathrm{BSM}}|$, in the models considered in the text. The weak phase of h_{BSM} is taken to vary in the entire range 0–2π. We also give the allowed values of the lifetime differences and CP phase alongwith the decay modes whose rates have strong correlations with the enhancement of the lifetime difference.

- Among the models considered here, only the third generation leptoquark model predicts a large enhancement of $\Delta\Gamma_q/\Gamma_q$ over the SM range. In this model, the value of $\Delta\Gamma_s/\Gamma_s$ can be as high as 0.51, which is restricted by the current experimental 95% C.L. bound from direct measurements. Improvements in the $\Delta\Gamma_s/\Gamma_s$ measurements will hence either detect new physics of this kind, or will bound the coupling product $\lambda_{32}^*\lambda_{33}$ in this model.

- An enhancement of a factor of up to 5 is also possible for $\Delta\Gamma_d/\Gamma_d$ in the models with the third generation leptoquark. Whereas the SM prediction for this quantity is $\sim 0.4\%$, making its measurement extremely difficult and rather unappealing, $\Delta\Gamma_d/\Gamma_d \sim 2.5\%$ is within the capability of the B factories. Limiting the value of $\Delta\Gamma_d/\Gamma_d$ from above also translates to bounding the coupling product $\lambda_{31}^*\lambda_{33}$ in this model.

- The enhancement in $\Delta\Gamma_q/\Gamma_q$ is correlated with an enhancement of $B_q \to \tau^+\tau^-$. Thus, looking for τ pairs in BaBar, Belle, and LHCb is of major importance. With the current constraints on all the parameters in the third generation leptoquark model, the branching ratio of $B_s \to \tau^+\tau^-$ ($B_d \to \tau^+\tau^-$) can be as high as 18% (0.4%). Though an extremely difficult measurement, the gains from the analysis of this decay

are significant.

- The mixing-induced phase β_s of the B_s–$\overline{B}_s$ system can be large with the third generation leptoquark as well as with the RPV SUSY: $|\beta_s|$ can become as high as 45° in the leptoquark model and up to 20° in RPV SUSY. Currently there is almost no constraint on β_s, but improved measurements, if they do not detect new physics through CP-violating signals in $B_s \to J/\psi\phi$, $B_s \to D_s^{+(*)}D_s^{-(*)}$, or $B_s \to J/\psi\eta^{(\prime)}$, can further squeeze the parameter space for BSM physics.

- Clearly, while non-MFV models are essential for enhancing $\Delta\Gamma_q/\Gamma_q$, not all of them serve the purpose. Among the models we have considered, while the third generation leptoquark models give large $\Delta\Gamma_q/\Gamma_q$ enhancement, the second generation leptoquark models fail to do so because of severe constraints on their parameter space. The RPV SUSY model, on the other hand, does not have as tightly constrained couplings, but the structure of the new physics contributions is such that though β_s can be enhanced, the value of $\Delta\Gamma_q/\Gamma_q$ cannot be increased. The three scenarios considered by us thus represent three different facets of non-MFV models.

Our analysis uses naive factorisation, and computes Γ_{12q} only to leading order. In RPV SUSY, we neglect the renormalisation group running of the Wilson coefficients and the mixing of the RPV operator with the others. However, since we claim $\Delta\Gamma_q/\Gamma_q$ enhancements of up to a factor of 5, and since there are anyway uncertainties in the model predictions due to the unknown masses of leptoquarks or squarks, improved calculations that take care of the above lacunae are not expected to change the conclusions qualitatively. We also take the strong phase difference between the SM and the BSM contributions to the hadronic decays to be vanishing. However, since the CP violation in hadronic decays does not play any role in our analysis, and we fit the hadronic decay form factors to the measured rates, this assumption has no impact on the predicted enhancement of $\Delta\Gamma_q/\Gamma_q$.

Large values of $\Delta\Gamma_q$ are possible only in a special class of models that have flavour dependent couplings and light degrees of freedom in the B_q–$\overline{B}_q$ mixing box diagram. In addition, the enhancement in $\Delta\Gamma_q/\Gamma_q$ comes coupled with an enhancement in some correlated decay rates and CP asymmetries. Hence, a measurement of large $\Delta\Gamma_q$ would point one towards very specific sources of new physics. Efforts towards the measurement of $\Delta\Gamma_s$ and $\Delta\Gamma_d$, as

well as the decay rates like $B_{s/d} \to \tau^+\tau^-$ are therefore highly encouraged; they may open the door to a rich phenomenology.

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